Data Structures and Algorithms

(**CS210A**)

Semester I – **2014-15**

Lecture 41

- **Miscellaneous problems**

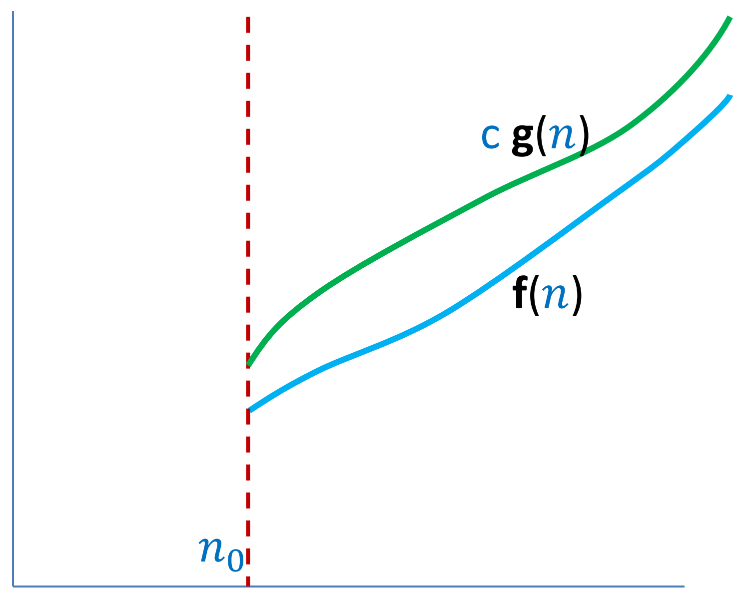
Order notation

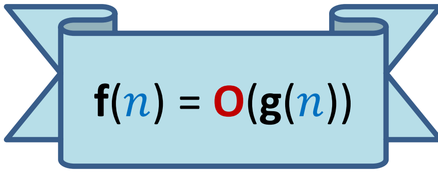
Definition: Let **f**(n) and **g**(n) be any two increasing functions of n.

f(n) is said to be <u>of the order of</u> **g**(n)

if there exist constants c and n_0 such that

$$f(n) \le c\, g(n) \quad \text{for all } n > n_0$$





Order notation extended

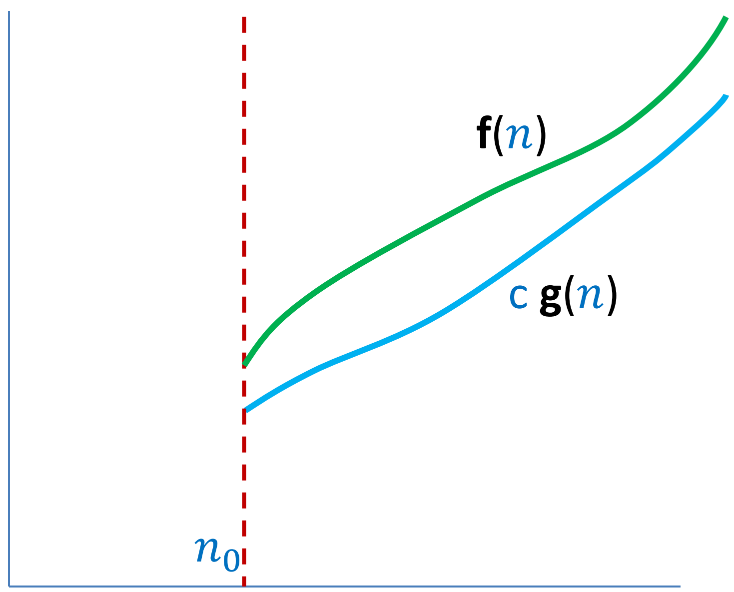
Definition: Let **f**(n) and **g**(n) be any two increasing functions of n.

f(n) is said to be lower bounded by **g**(n)

if there exist constants c and n_0 such that

$$f(n) \geq c\, g(n) \quad \text{for all } n > n_0$$

f(n) = **Ω**(**g**(n))



$$\frac{n^2}{100} = \Omega(10000\, n \log n)$$

Order notation extended

Observations:

- **f**(n) = **O**(**g**(n)) **if and only if** **g**(n) = **Ω**(**f**(n))

One more Notation:

If **f**(n) = **O**(**g**(n)) **and** **g**(n) = **O**(**f**(n)), then

g(n) = **Θ**(**f**(n))

Examples:

- $\frac{n^2}{100} = \Theta(10000\, n^2)$

- Time complexity of Quick Sort is $\Omega(n \log n)$

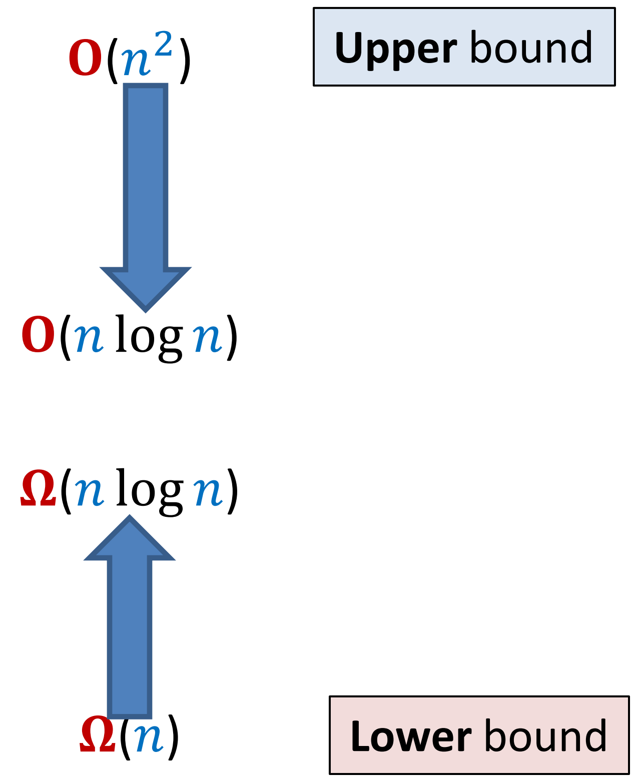
- Time complexity of Merge sort is $\Theta(n \log n)$

Time complexity of a problem

Example: Sorting

- **Algorithm 1** : Selection Sort with time complexity $\mathbf{O}(n^2)$
- **Algorithm 2** : Merge Sort with time complexity $\mathbf{O}(n \log n)$
- Each comparison based sorting algorithm need to perform $\boldsymbol{\Omega}(n \log n)$ comparisons in the worst case.
- Sorting must takes $\boldsymbol{\Omega}(n)$ time since it has to read each item at least once.

Time complexity of sorting



Sorting has time complexity of $\boldsymbol{\Theta}(n \log n)$

Time complexity of a problem

Example: All-pairs shortest paths (APSP)

- **Algorithm 1** : Floyd Warshal Algorithm with time complexity $\mathbf{O}(n^3)$
- **Algorithm 2** : Johnson' algorithm with time complexity $\mathbf{O}(mn \log n)$
- All-pairs shortest paths must require $\mathbf{\Omega}(n^2)$ time

Time complexity of APSP

$\mathbf{O}(n^3)$ — **Upper** bound

↓

$\mathbf{O}(mn \log n)$

$\mathbf{\Omega}(n^2)$ — **Lower** bound

There is still a gap between upper and lower bounds for APSP. ☹

Aim of theoretical computer science

For any given computational problem ***P***

- Get **smallest possible upper bound** on its time complexity

This requires designing better algorithm

Reduce the GAP

- Get **largest possible lower bound** on its time complexity.

?

How to establish lower bound

Two ways:

- **Adversarial approach**

  A gentle introduction today

- **Limitation** of the model of computation

  CS345

Adversarial approach

Key aspects

- Algorithm does not have free access to the input. To access any item in the input, algorithm has to spend some time.
- The execution of an algorithm at any step is fully determined **only** by the (partial) input it has seen till now.
- Adversary has access to all possible inputs of a problem.
- The sole aim of adversary is to make an algorithm work really hard. For this purpose, adversary discloses the input *cleverly*.

Locating 1 problem

Input: An array **A**$[0 \ldots n-1]$ with an unknown i s.t.

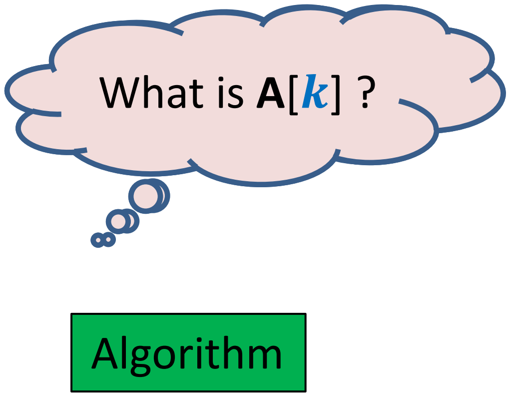
- For all $j \neq i$, **A**[j]$= 0$
- **A**[i]$= 1$

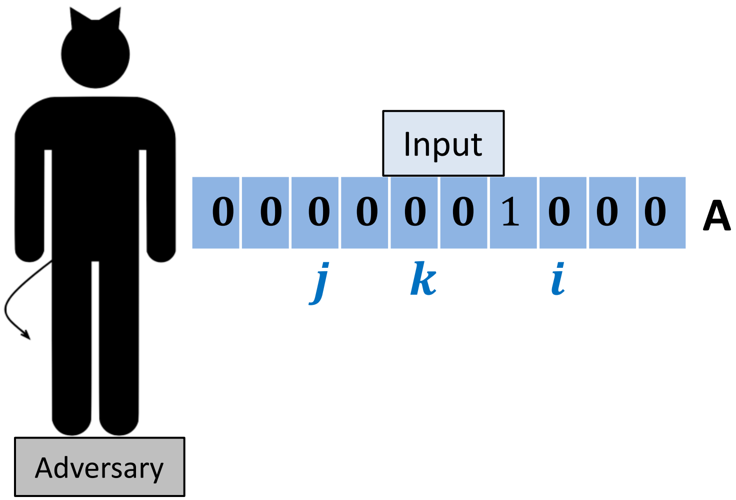
Aim: To locate/search 1 in **A**.

Upper bound: **O**(n)

Lower bound: Ω(n)

Lower bound on Locating 1 problem





Miscellaneous problems

Problem 1

Input:

Given an array **A** storing n numbers,

there is an $i < n$ (unknown) s.t.

A$[0] <$ **A**$[1] <$ …$<$ **A**$[i]$ $>$ **A**$[i+1] >$**…** $>$**A**$[n-1]$

Aim:

To search efficiently

Answer : **O**(log n) is possible

Problem 1

Input: Given an array **A** storing n numbers,

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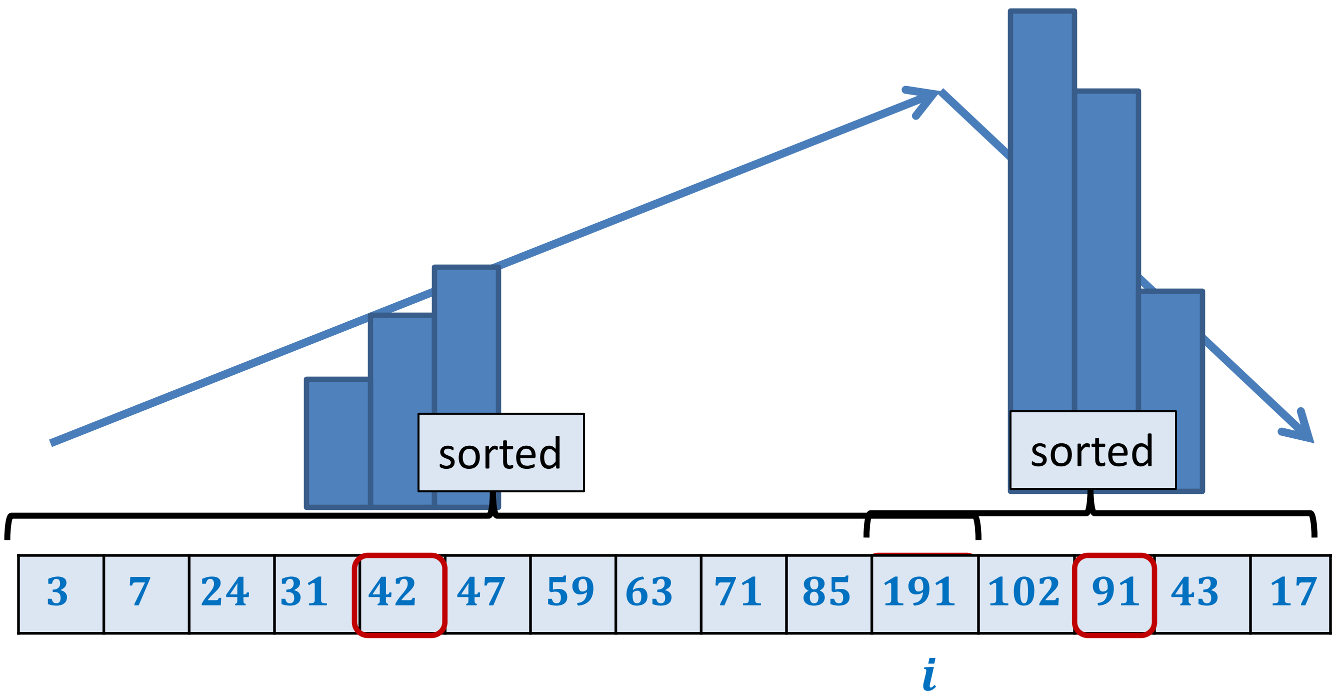


Problem 2

Input:

Given an array **A** storing n numbers,

there is an $i < n$ (unknown) s.t.

A$[0] \leq$ **A**$[1] \leq \ldots \leq$ **A**$[i]$ $\geq$ **A**$[i+1] \geq \ldots \geq$ **A**$[n-1]$

Aim:

To search efficiently

Answer : $\Omega(n)$ time complexity

Locating 1 problem is a special case of **Problem 2**

Problem 3

Input:

Given an array **A** storing n numbers,

there are $i < j < n$ (unknown) s.t.

A$[0] <$ **A**$[1] <$ …$<$ **A**$[i]$ $>$ **A**$[i+1] >$**…** $>$**A**$[j]$ $<$ **A**$[j+1] <$**…** $<$**A**$[n-1]$

Aim:

To search efficiently

Answer : $\Omega(n)$ time complexity

Locating 0 problem

Input: An array **A**$[0 \ldots n-1]$ with an unknown i s.t.

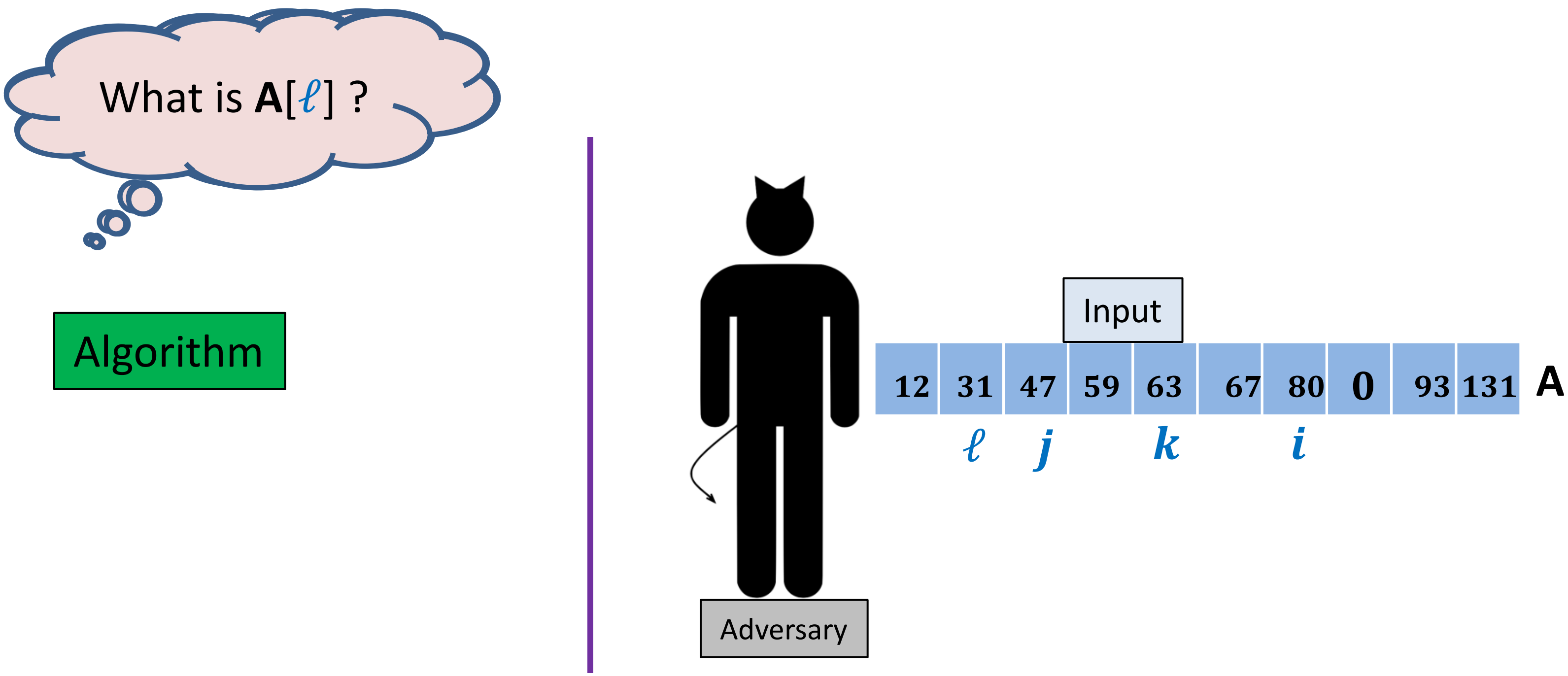
- For all $j \neq i$, **A**$[j] > 0$ **and** **A**$[j] <$ **A**$[j+1]$
- **A**$[i]$ = $\mathbf{0}$

Aim: To locate/search **0** in **A**.

Upper bound: **O**(n)

Lower bound: $\Omega(n)$

Lower bound on Locating 0 problem



Problem 3

Input:

Given an array **A** storing n numbers,

there are $i < j < n$ (unknown) s.t.

A$[0]<$ **A**$[1]<$ …$<$ **A**$[i]$ $>$ **A**$[i+1]$ $>$**…** $>$**A**$[j]$ $<$ **A**$[j+1]$ $<$**…** $<$**A**$[n-1]$

Aim:

To search efficiently

Answer : $\Omega(n)$ time complexity

Locating 0 **problem** is a special case of **Problem 3**

Final slide

That’s all. I hope you enjoyed this lecture.