CS 203B: Mathematics for Computer Science-III
Assignment 5

Deadline: 18 : 00 hours, September 11, 2015

General Instructions:

- Write your solutions by furnishing all relevant details (you may assume the results already covered in the class).
- You are strongly encouraged to solve the problems by yourself.
- You may discuss but write the solutions on your own. Any copying will get zero in the whole assignment.
- If you need any clarification, please contact any one of the TAs.
- Please submit the assignment at KD-213/RM-504 before the deadline. Delay in submission will cause deduction in marks.

Question 1: [5+5] Prove that:

(a) the ring $\mathbb{Q}[\sqrt{d}] = \mathbb{Q}(\sqrt{d})$ and therefore a field for any $d \in \mathbb{Z}$,

(b) the field $\mathbb{Q}[e^{\frac{2\pi i}{3}}]$ is isomorphic to $\mathbb{Q}[x]/(x^2 + x + 1)$.

Question 2: [5+5]

Let F and K be fields such that $F \subseteq K$. Field K is called an *extension of F of degree d* if K is isomorphism to $F[x]/(f(x))$ with polynomial $f(x)$ of degree d. The fields in the previous questions are all extensions of $\mathbb{Q}$ of degree 2. Give example of a field, contained in $\mathbb{C}$, of degree 3 over $\mathbb{Q}$. Extend the construction to any degree $d > 0$.

Question 3: [10]

An algebraic number a is said to be an *algebraic integer* if it satisfies an equation of the form $a^m + \alpha_1 a^{m-1} + \cdots + \alpha_m = 0$, where $\alpha_1, \cdots, \alpha_m$ are integers. Now suppose β is an algebraic integer satisfying $\beta^3 + \beta + 1 = 0$ and γ is an algebraic integer satisfying $\gamma^2 + \gamma - 3 = 0$. Show that both $\beta + \gamma$ and $\beta\gamma$ are also algebraic integers.

Question 4: [10]

A real number α is said to be a *constructible* if by the use of straightedge and compass alone we can draw a line segment of length α, assuming that we are given some fundamental unit length.

Prove that if α, β are constructible, then so are $\alpha + \beta$, $\alpha - \beta$, $\alpha\beta$ and α/β (when $\beta \neq 0$). Therefore, the set of constructible numbers forms a subfield, W, of the field of real numbers. It is easy to check that W contains the field of rational numbers.