CS 203B: Mathematics for Computer Science-III
Assignment 1

Deadline: 18 : 00 hours, August 12, 2015

General Instructions:

- Write your solutions by furnishing all relevant details (you may assume the results already covered in the class).
- You are strongly encouraged to solve the problems by yourself.
- You may discuss but write the solutions on your own. Any copying will get zero in the whole assignment.
- If you need any clarification, please contact the instructor or any one of the TAs.
- Please submit the assignment at KD-213/RM-504 before the deadline. Delay in submitting will cause deduction in marks.

Question 1: [3+2+5+5]

For $n \geq 3$, consider a regular polygon with n-sides. Number the vertices $1, \cdots, n$ in the counterclockwise direction. Let r be the rotation through $2\pi/n$ about the center of polygon (so $i \longmapsto i+1 \bmod n$), and let s be the reflection in the line (rotation about the line) through the vertex 1 and the center of the polygon (so $i \longmapsto n+2-i \bmod n$). Also let e be the identity map on the vertices set. Now prove the following:

1. $e, r, \cdots, r^{n-1}$ are all distinct and $r^n = e$.
2. $s^2 = e$.
3. $D_{2n} = \{e, r, \cdots, r^{n-1}, s, sr, sr^2, \cdots, sr^{n-1}\}$ has size $2n$.
4. If x is any element of D_{2n}, which is not a power of r, then show that $rx = xr^{-1}$.

Question 2: [10]

A *graph* is an ordered pair $G = (V, E)$ comprising a set V of vertices together with a set E of edges which are 2-element subsets of V, i.e., $E \subseteq \{(u, v) | u, v \in V\}$.

An *automorphism* of a graph $G = (V, E)$ is a permutation σ on the vertex set V, such that the pair of vertices $(u, v) \in E$ if and only if the pair $(\sigma(u), \sigma(v)) \in E$. Now consider the set of all such automorphisms and let us denote this set as $Aut(G)$. Prove that $Aut(G)$ forms a group under *composition operation*. Suppose $\sigma : V \to V$ and $\tau : V \to V$, then *composition operator* is defined as $\sigma \circ \tau : V \to V$ such that for any $v \in V$, $\sigma \circ \tau(v) = \sigma(\tau(v))$.

Question 3: [10]
Consider the following set

$$S = \{(x, y)|x, y \in \mathbb{R}, x^2 + y^2 = 1 \text{ and } y < 0\}.$$

Can you think about an operation defined on the elements of S such that S forms a group under that operation?

Question 4: [8+7]

Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are integers modulo p, for a prime p such that $ad - bc \neq 0$. Prove that G forms a group under matrix multiplication. What is the order of G?

Let H be a subset of G defined by

$$H := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G | ad - bc = 1 \right\}.$$

Show that H is a subgroup of G. What is the order of H?

Question 5: [10]

Prove that D_{24} (group defined in Question 1) and S_4 (symmetric group of order 24) are not isomorphic.