CS202A Final Exam

February 17, 2015

Name ....

Roll No: ....

Maximum marks:70
Duration 2 hours

Closed books and Notes

Instructions

1. This is question paper cum answerbook. Answers to all questions should be written only in the space provided after the question.

2. Only final answers should be written in the space provided for answers. Further, these should be written neatly and in the format asked.

3. No rough work should be done in the space provided to answer the questions. Space for rough-work is given at the end of the this answerbook. Additional rough sheets may also be requested.

4. In your Natural Deduction (ND) proofs you are allowed to use any propositional entailment in single step, without proving it.

5. You may also use four derived rules of propositional ND.

6. Our notation $A(x)$ for a first order formula implies that A has no free variable other than x. Similar convention holds for $B(y)$, $C(z)$ etc.

Q1 Give Natural Dedction proofs of the following. Show clearly the boxes used, if any, and write justification against each line in your proof.

(a) $\forall x \forall y (A(y) \rightarrow B(x)) \vdash \exists y A(y) \rightarrow \forall x B(x)$ [marks 5]

(b) $\forall x(A(x) \to B(x) \vee C(x)), \neg \exists x(A(x) \wedge C(x)) \vdash \forall x(A(x) \to B(x))$

[marks 5]

(c) $\forall x\phi(x) \to \psi \vdash \exists x(\phi(x) \to \psi)$, where $x \notin free(\psi)$.

[marks 8]

Q2 (a) Convert the following formula in prenex normal form.

$\forall x \exists y (A(x) \to B(y)) \to \exists y \forall x (A(x) \to B(y))$ [marks 4]

(b) Is the formula given, in part **(a)**, valid? Prove your answer.
[You may either use result of part **(a)** or do it directly]. [marks 4]

Q3 Consider the following formulae.

1. $\forall x \neg R(x, x)$
2. $\forall x \forall y \forall z (R(x, y) \wedge R(y, z) \rightarrow R(x, z))$
3. $\forall x \exists y R(x, y)$

(a) Show a model of these formulae. (That is, show a structure (A, R^A), in which all these formulae are true).

[marks 2]

(b) Argue clearly that every model of these formulae has infinite domain.

[marks 5]

Q4 Consider a possible world frame (W, R) in which

$$\forall u \in W \forall v \in W [R(u, v) \vee R(v, u)].$$

(a) Show that $(W, R) \models \Box(\phi \wedge \Box\phi \to \psi) \vee \Box(\psi \wedge \Box\psi \to \phi)$. [marks 5]

(b) Also show an example of a possible world model with frame of the above kind s.t. $(W, R) \not\models \Box(\phi \wedge \Box\phi \to \psi) \vee \Box(\psi \to \phi)$. [marks 2]

Q5 Give a Natural Dedction proof of the following.

$\Box p \to \Box q \vdash_{KT45} \Box(\Box p \to \Box q)$

Show clearly the boxes used, if any, and write justification against each line in your proof.

[**Hint:** you may like to use LEM]

[marks 10]

Q6 Consider the Knowledge model with set of worlds $W = \{x_1, \ldots, x_6\}$ and with three agents $\{A, B, C\}$ as shown below. Note that R_A, R_B and R_C are equivalence relations.

In each of the parts **(a)-(c)** below, find the set of worlds where the given formula holds. Briefly justify your answers in each case.

(a) $K_A(r)$ holds [marks 3]

(b) $K_B \neg K_A(r)$ holds [marks 3]

(c) $C_G(\neg q)$ holds, where $G = \{A, C\}$. [marks 4]

Q7 Consider a scenario of muddy children puzzle where $n > 3$ and children $1, 2$ and 3 have mud on their face. As usual, let p_i denote that child i has mud on her face.

Let $\alpha = \bigwedge \{C(p_i \to K_j(p_i)) \mid 1 \leq i, j \leq n,\ i \neq j\}$.

(a) Show that

$$\alpha,\ p_3 \vdash_{KT45^n} K_1 K_2(p_3)$$

[marks 6]

(b) Argue semantically that in the given scenario, even before mother's statement, $\bigwedge_{l=1}^{n} \bigwedge_{m=1}^{n} K_l K_m((\bigvee_{i=1}^{n} p_i)$. [marks 2]

(c) From part (b), we know that even before mother's statement, every child knows that every child knows that there is a child with mud on her face. What is changed by mother's statement? Justify your answer. [marks 2]

Rough Work

Rough Work

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