**CS202A** Assignment 2

November 04, 2016.
*Due on:* 11 November, 2016. Please hand over your assignment to the TA in class.

1. Using the letters indicated for predicates and constants, translate the following natural language sentences into the language of first order logic. The variables following the predicates are not necessarily the ones you should use in sybolizing the sentences.
   (a) *Some students who do not like any language courses prefer both physics and mathematics to chemistry.* (S(x): x is a student, L(x,y): x likes y, V(x): x is a language course, P(x,y,z): x prefers y to z, p: physics, m: mathematics, c: chemistry.)
   (b) *There is no instant of time such that every instant of time is after it.* (I(x): x an instant of time, A(x,y): x is after y.)

2. Use appropriate predicate symbols to symbolize the argument below. Then give a natural deduction proof of correctness of the argument.

   *None of Ockham's followers likes any realist. Any of Ockham's followers likes at least one of Hobbes' followers. Moreover, Ockham does have followers. Therefore, some of Hobbes' followers are not realists.*

3. Prove the validity of

   (a) $t_1 = t_2 \vdash (t + t_2) = (t + t_1)$

   (b) $\neg \exists x P(x) \vdash \forall x \neg P(x)$

   (c) $\forall x \neg P(x) \vdash \neg \exists x P(x)$

   (d) $\exists x(\phi \rightarrow \psi) \vdash \forall x \phi \rightarrow \psi$, where $\psi$ has no free occurrences of $x$.

4. Give a formula of first order predicate calculus $\phi$ such that $\phi$ evaluates to true in a model iff the model has exactly three distinct elements in its universe.

5. Following are two sequents (a) and (b), one of which is valid and the other is not. Identify the one which is valid, give a natural deduction proof of the sequent. Prove that the other one is not valid.
   (a) $\forall x(P(x) \vee Q(x)) \vdash \forall x P(x) \vee \forall x Q(x)$
   (b) $\forall x P(x) \vee \forall x Q(x) \vdash \forall x(P(x) \vee Q(x))$