CS202A Assignment 1 (Propositional Logic)

Submission Deadline: Nov 2, 3pm

Instructions

- You are expected to submit assignment by submission deadline mentioned above. Solutions to all questions will be posted on the course website soon after the submission deadline.
- You are expected to submit answers worked out with paper and pen and written in your own handwriting. Write your answers neatly, to the point. Strike out all your rough work.
- While you are expected to answer all the questions, to keep the grading effort manageable we will check only a subset (may be two) of these questions. The questions to be graded will be selected in somewhat random fashion after the submission deadline. Same questions will be graded for all students.

Q1

(a) Prove the following in Natural deduction without using derived rules.

(i) $\neg(p \vee q) \dashv\vdash \neg p \wedge \neg q$

(ii) $\neg(p \wedge q) \dashv\vdash \neg p \vee \neg q$

(iii) $\neg q \rightarrow \neg p \vdash p \rightarrow q$

(b) Prove the following in Natural deduction possibly using derived rules.

(i) $\neg(p \rightarrow q) \vdash q \rightarrow p$

(ii) $(p \to q) \to q \vdash (q \to p) \to p$

Q2 In this question you have to solve Q6, Exercise 1.2 of the textbook. Read this question from the textbook.

As an additional example. rule $\wedge i$ becomes the inference

$\frac{\Gamma_1 \vdash \phi_1 \quad \Gamma_2 \vdash \phi_2}{\Gamma_1 \cup \Gamma_2 \vdash \phi_1 \wedge \phi_2} \quad \wedge i$

Answer the following.

(a) As in the textbook.

(b) Proofs is this new system can be written both in linear form as well as in tree form as is the case for ND. Do you need boxes when writing proofs in linear form in the new system? Why or why not?

(c) Prove $\vdash p \vee \neg p$ in the new system. [Hint: first construct an ND proof, then associate a sequent with each line of this proof]

(d) Can you prove $p \vdash q \to p$ in the new system? Justify your answer.

Q3

(a) Let $G = (V, E)$ be an undirected graph. G is 4 colorable iff V can be partitioned into four (disjoint) sets V_1, V_2 V_3 and V_4, each set thought of as a distinct color, s.t. for all edges e in G both end points of e have different colors. Show that for every graph G there is a propositional formula ϕ_G which is satisfiable iff G is 4 colorable. Further size of ϕ_G is bounded by a polynomial in size of G and ϕ_G can be constructed from G in polynomial time. [Hint: assign four propositional variables p_v^1, p_v^2, p_v^3 and p_v^4 to each vertex v of G, p_v^i is true iff vertex v is colored with color i.]

(b) Similar to part (a), show that given a directed graph $G = (V, E)$ and two vertices $u, v \in V$ the problem of finding if there is a path from u to v in G can be reduced to satisfiability checking. (you are not allowed to do any non-trivial computation on graph before producing a propositional formula for it).

Q4 Consider rules of the following form.

(i) $p \to q_1 \vee q_2 \vee \ldots \vee q_n$, $n \geq 0$

(ii) $\rightarrow q_1 \vee q_2 \vee \ldots \vee q_n$

where p, q_i are positive literals. In particular, each rule has at most one literal on the left of the implication.

Using the ideas in Horn clause satisfiability problem done in class, design an efficient algorithm to check satisfiability of a set of rules of the form above.

Q5 **(a)** Show that the second SAT solver considered in class terminates in cubic time.

(b) Design a SAT instance on which above SAT solver fails (that is it cannot decide if the given instance is satisfiable or unsatisfiable). Prove your answer.

(c) How does our linear SAT solver work on a set of Horn clauses? How does the cubic SAT solver work? Justify your answers. You need to say when will the solver succeed or fail.

[Hint: try a simple example].

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